Some Standard Elementary Integrals

Based on differentiation and definition of integration, we have the following standard results. The student is strongly advised to commit these results to memory, because no further progress is otherwise possible.

(i) $\frac{d}{dx}(c) = 0 \Rightarrow \int 0 . dx = c$

(ii) $\frac{d}{dx}(x) = 1 \Rightarrow \int 1 . dx = x + c$

(iii) $\frac{d}{dx}(kx) = k \Rightarrow \int k\, dx = kx + c$

(iv) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{(n+1)} . (n+1)x^n = x^n ; n \neq -1$

$\Rightarrow \int x^n . dx = \frac{x^{n+1}}{n+1} + c ; n \neq -1$

(v) $\frac{d}{dx}[\log | x |] = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \log | x | + c$

(vi) $\frac{d}{dx}(e^x) = e^x \Rightarrow \int e^x\, dx = e^x + c.$

(vii) $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = \frac{a^x (\log a)}{(\log a)} = a^x ; a > 0 ; a \neq 1$

$\Rightarrow \int a^x\, dx = \frac{a^x}{\log a} + c ; a > 0, a \neq 1$

(viii) $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\, dx = \sin x + c$

(ix) $\frac{d}{dx}(-\cos x) = \sin x \Rightarrow \int \sin x\, dx = -\cos x + c$

(x) $\frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \int \sec^2 x\, dx = \tan x + c$

(xi) $\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x \Rightarrow \int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(xii) $\frac{d}{dx}(\sec x) = \sec x \tan x \Rightarrow \int \sec x \tan x\, dx = \sec x + c$

(xiii) $\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x \Rightarrow \int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(xiv) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

(xv) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \Rightarrow \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

(xvi) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Note. We know that $\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

From this result, it should not be concluded that $\sin^{-1} x = -\cos^{-1} x$. Rather, $\sin^{-1} x$ and $\cos^{-1} x$ differ by a constant.

(xvii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \Rightarrow \int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$

(xviii) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2} \Rightarrow \int \frac{-1}{1+x^2} dx = \cot^{-1} x + c$

(xix) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}} \Rightarrow \int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c.$